79 Horizontal, Compound, Vertical, and Spiral Curves

Nomenclature

A	absolute value of the algebraic grade difference	percent	percent
C	chord length	ft	m
C	clearance	ft	m
D	degree of curve	deg	deg
e	superelevation rate	ft/ft	m/m
E	equilibrium elevation	ft	m
E	external distance	ft	m
f	coefficient of friction	–	–
F	force	lbf	N
g	acceleration due to gravity	ft/sec^2	m/s^2
G	gauge	ft	m
G	grade	decimal	decimal
h	height	ft	m
I	interior angle	deg	deg
K	length of vertical curve per percent grade difference	ft/%	m/%
l	length	ft	m
L	length of curve	ft	m
L	superelevation runoff	ft	m
m	mass	lbm	kg
M	middle ordinate	ft	m
p	cross slope (rate)	ft/ft	m/m
R	curve radius	ft	m
S	sight distance	ft	m
SRR	superelevation runout rate	ft/ft	m/m
t	time	sec	s
T	tangent length	ft	m
v	velocity	ft/sec	m/s
W	lane width	ft	m
W	offset (maximum)	ft	m
x	distance from BVC	sta	sta
x	tangent distance	ft	m

Symbols

α	angle	deg	deg
γ	angle	deg	deg
θ	angle	deg	deg
ϕ	angle	deg	deg

Subscripts

c	centrifugal or circular curve
eff	effective
f	frictional
p	perception-reaction
R	runout
s	side friction or spiral
t	tangential

1. HORIZONTAL CURVES

A *horizontal circular curve* is a circular arc between two straight lines known as *tangents*. When traveling in a particular direction, the first tangent encountered is the *back tangent (approach tangent)*, and the second tangent encountered is the *forward tangent (departure tangent)*.

The geometric elements of a horizontal circular curve are shown in Fig. 79.1. Table 79.1 lists the standard terms and abbreviations used to describe the elements. Equations 79.1 through 79.7 describe the basic relationships between the elements. The *interior angle*, I, has units of degrees unless indicated otherwise. (Equations that contain the *degree of curve* term, D, are only to be used with customary U.S. units.)

$$R = \frac{5729.578}{D} \quad \text{[U.S.—arc definition]} \qquad 79.1$$

$$R = \frac{50}{\sin\left(\frac{D}{2}\right)} \quad \text{[U.S.—chord definition]} \qquad 79.2$$

$$L = \frac{2\pi RI}{360^\circ} = RI_{\text{radians}} = 100\left(\frac{I}{D}\right) \quad \text{[U.S.]} \qquad 79.3$$

$$T = R\tan\left(\frac{I}{2}\right) \qquad 79.4$$

$$E = R\left(\sec\left(\frac{I}{2}\right) - 1\right) \qquad 79.5$$

$$M = R\left(1 - \cos\left(\frac{I}{2}\right)\right) = \left(\frac{C}{2}\right)\tan\left(\frac{I}{4}\right) \qquad 79.6$$

$$C = 2R\sin\left(\frac{I}{2}\right) = 2T\cos\left(\frac{I}{2}\right) \qquad 79.7$$

Table 79.1 *Horizontal Curves: Abbreviations and Terms*

preferred	
c	short chord (any straight distance from one point on the curve to another)
C	long chord (chord PC to PT); same as LC
D	degree of curve
E	external distance (the distance from the vertex to the midpoint of the curve)
I	interior angle; central angle of curve; deflection angle between back and forward tangents
L	length of the curve (the length of the curve from the PC to the PT)
LC	long chord (chord PC to PT); same as C
M	middle ordinate (the distance from the curve midpoint to the midpoint of the long chord)
MPC	midpoint of curve
PC	point of curvature (the point where the back tangent ends and the curve begins)
PI	point of intersection of back and forward tangents
POC	(any) point on the curve
POCT	point of curve tangent
POST	(any) point on the semitangent
POT	(any) point on the tangent
PT	point of tangency (the point where the curve ends and the forward tangent begins)
R	radius of the curve
RP	radius point (center of curve)
T	(semi-) tangent distance from V to PC or from V to PT
V	vertex of the tangent intersection point
alternate/archaic	
BC	beginning of curve (same as PC and TC)
CT	change from curve to tangent (same as PT and EC)
Δ	interior angle (same as I)
EC	end of curve (same as PT and CT)
PVI	point of vertical intersection (same as PI)
TC	a change from a tangent to a curve (same as BC and PC)

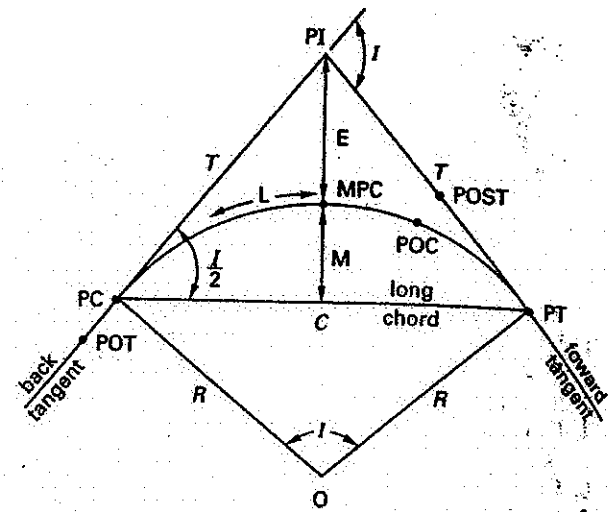
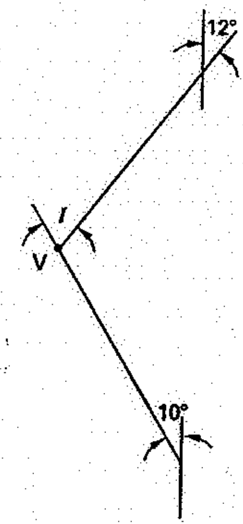


Figure 79.1 *Horizontal Curve Elements*

Example 79.1

Two tangents—the first entering a horizontal curve and the second leaving the horizontal curve—have bearings of N10°W and N12°E, respectively. The curve radius is 1300 ft. Determine the interior angle.



Solution

Since the two tangents do not originate fro[m] ... point, the interior angle is the deflection ang[le] ... the two tangents.

$$I = 10^\circ + 12^\circ = 22^\circ$$

2. DEGREE OF CURVE

In the United States, curvature of city st... ways, and railway curves can be specified b[y] ... radius, R, or the *degree of curve*, D. There... lel concept in metric highway design. There... are only denoted by radius when using me[tric] ...

$$D = \frac{(360^\circ)(100)}{2\pi R} = \frac{5729.578}{R} \quad \text{[arc ...}$$

highway work, the *length of the curve* is under- be the actual curved arc length, and the degree urve is the angle subtended by an arc of 100 ft. he degree of curve is related to an arc of 100 ft, d to be calculated on an *arc basis*.

y curves have very large radii. The radii are so hat short portions of the curves are essentially t. In railroad surveys, the *chord basis* is used, e degree of curve has a different definition. The d degree of curve is the angle subtended by a of 100 ft. In that case, the degree of curve and are related by Eq. 79.9. Once the radius is cal- d, other arc-basis equations (Eqs. 79.1 through can be used.

$$\sin\left(\frac{D}{2}\right) = \frac{50}{R} \quad \text{[chord basis]} \qquad 79.9$$

re the radius is large (4° curves or smaller), the rence between the arc length and the chord length ignificant. Therefore, the length of the curve in oad practice is equal to the number of 100 ft chords.

$$L \approx \left(\frac{I}{D}\right)(100 \text{ ft}) \qquad 79.10$$

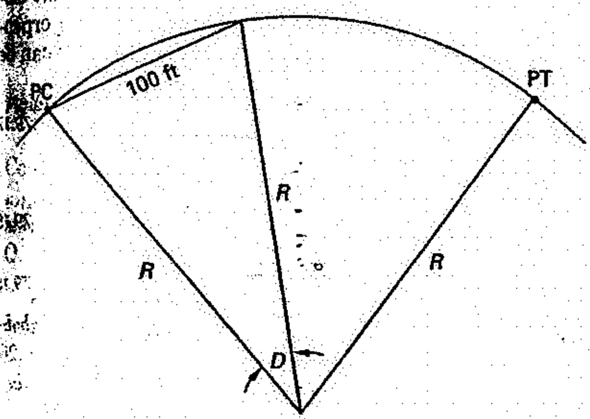


Figure 79.2 Horizontal Railroad Curve (chord basis)

3. STATIONING ON A HORIZONTAL CURVE

When the route is initially laid out between PIs, the curve is undefined. The "route" distance is measured from PI to PI. The route distance changes, though, when the curve is laid out. Stationing along the curve is continuous, as a vehicle's odometer would record the distance. The PT station is equal to the PC station plus the curve length.

$$\text{sta PT} = \text{sta PC} + L \qquad 79.11$$

$$\text{sta PC} = \text{sta PI} - T \qquad 79.12$$

When the curve is laid out, each PI will have two stations associated with it. The *forward station* is equal to the PC station plus the tangent length. The *back station* is equal to the PT station minus the tangent length.

Example 79.2

An interior angle of 8.4° is specified for a 2° horizontal curve. The forward PI station is 64+27.46. Locate the PC and PT stations.

Solution

Use Eq. 79.8.

$$R = \frac{(360°)(100 \text{ ft})}{2\pi D} = \frac{(360°)(100 \text{ ft})}{(2\pi)(2°)} = 2864.79 \text{ ft}$$

Use Eqs. 79.4 and 79.3.

$$T = R\tan\left(\frac{I}{2}\right) = (2864.79 \text{ ft})\tan\left(\frac{8.4°}{2}\right) = 210.38 \text{ ft}$$

$$L = RI\left(\frac{2\pi}{360°}\right) = (2864.79 \text{ ft})(8.4°)\left(\frac{2\pi}{360°}\right) = 420.00 \text{ ft}$$

The PC and PT points are located at

$$\text{sta PC} = \text{sta PI} - T = (64{+}27.46) - 210.38 \text{ ft} = 62{+}17.08$$

$$\text{sta PT} = \text{sta PC} + L = (62{+}17.08) + 420.00 \text{ ft} = 66{+}37.08$$

4. CURVE LAYOUT BY DEFLECTION ANGLE

Construction survey stakes should be placed at the PC, at the PT, and at all full stations. (In the United States, 100 ft stations are the rule. Metric stationing may be 100 or 1000 m, depending on the organization.) Stakes may also be required at quarter or half stations and at all other critical locations.

The *deflection angle method* is a common method used for staking out the curve. A *deflection angle* is the angle between the tangent and a chord. Deflection angles are related to corresponding arcs by the following principles.

(1) The deflection angle between a tangent and a chord (Fig. 79.3(a)) is half of the arc's subtended angle.

(2) The angle between two chords (Fig. 79.3(b)) is half of the arc's subtended angle.

In Fig. 79.3(a), angle V-PC-A is a deflection angle between a tangent and a chord. Using Principle 1,

$$\alpha = \angle\text{V-PC-A} = \frac{\beta}{2} \qquad 79.13$$

Angle β can be found from the following relationships.

$$\frac{\beta}{360^\circ} = \frac{\text{arc length PC-A}}{2\pi R} \quad 79.14$$

$$\frac{\beta}{I} = \frac{\text{arc length PC-A}}{L} \quad 79.15$$

The chord distance PC-A is given by Eq. 79.16. The entire curve can be laid out from the PC by sighting the deflection angle V-PC-A and taping the chord distance PC-A.

$$C_{\text{PC-A}} = 2R\sin\alpha = 2R\sin\left(\frac{\beta}{2}\right) \quad 79.16$$

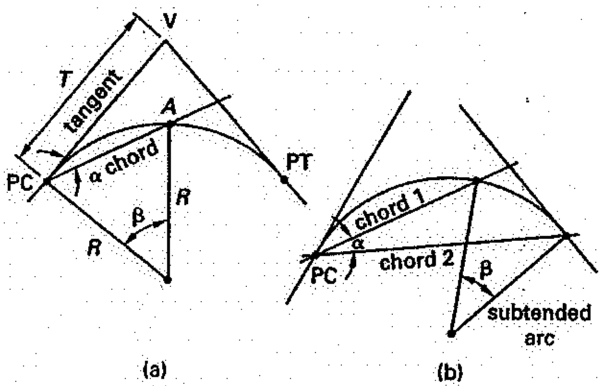


Figure 79.3 Circular Curve Deflection Angle

Example 79.3

A circular curve is to be constructed with a 225 ft radius and an interior angle of 55°. The separation between the stakes along the arc is 50 ft. (a) Determine the chord lengths between stakes. (b) Assuming the curve is laid out from point to point, specify the first and last deflection angles. (c) Determine the length of the final (partial) chord.

Customary U.S. Solution

(a) The central angle for an arc of 50 ft is given by Eq. 79.14.

$$\beta = \frac{(360^\circ)(\text{arc length})}{2\pi R} = \frac{(360^\circ)(50\text{ ft})}{2\pi(225\text{ ft})} = 12.732^\circ$$

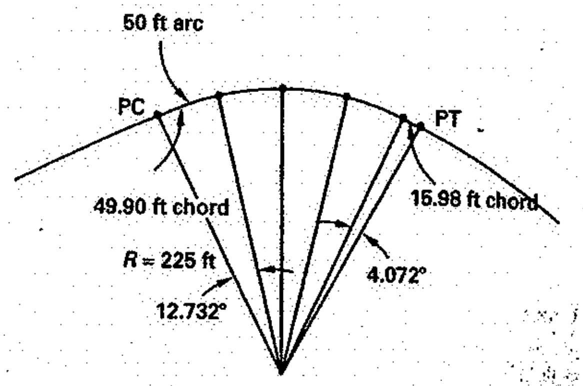
From Eq. 79.16, the required chord length for full 50 ft arcs is

$$C = 2R\sin\left(\frac{\beta}{2}\right) = (2)(225\text{ ft})\sin\left(\frac{12.732^\circ}{2}\right) = 49.90\text{ ft}$$

(b) The first central angle is 12.732°. From Principle 1, the first deflection angle is half of this or 6.366°.

12.732° goes into 55° four times with a remainder of 4.072°. The last deflection angle (sighting to the PT) is 2.036°. Use Eq. 79.16.

(c)
$$C = (2)(225\text{ ft})\sin\left(\frac{4.072^\circ}{2}\right) = 15.98\text{ ft}$$



5. TANGENT OFFSETS

A *tangent offset*, y, is the perpendicular distance from an extended tangent line to the curve. A *tangent distance*, x, is the distance along the tangent to a perpendicular point. Tangent offsets for circular curves can be calculated from Eq. 79.17.

$$y = R(1-\cos\beta) = R - \sqrt{R^2 - x^2}$$

$$\beta = \arcsin\left(\frac{x}{R}\right) = \arccos\left(\frac{R-y}{R}\right)$$

$$x = R\sin\beta = \sqrt{2Ry - y^2}$$

Offsets from *parabolic curves* (*parabolic flares, pa[illegible] tapers*, or *curb flares*) are calculated from Eq. [illegible] W is the maximum offset, L is the length of th[illegible] (*taper*), x is the distance along the baseline, a[illegible] the offset.

$$\frac{y}{W} = \frac{x^2}{L^2}$$

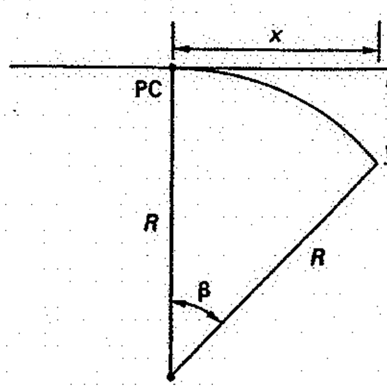


Figure 79.4 Tangent Offset

6. CURVE LAYOUT BY TANGENT OFFSETS

The *tangent offset method* (station offset method) can be used to lay out horizontal curves. This method is typically used on short curves. The method is named for the way in which the measurements are made, which is by measuring offsets from the tangent line.

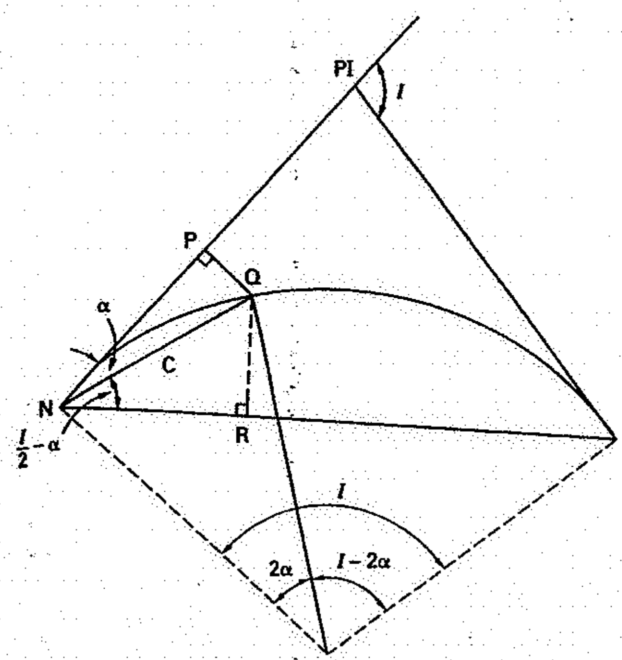


Figure 79.5 Tangent and Chord Offset Geometry

Consider the two right triangles NPQ and NRQ in Fig. 79.5. Point Q is any point along the circular curve, and α is the deflection angle from the tangent to point Q.

$$\text{NP} = \text{tangent distance} = \text{NQ}\cos\alpha \qquad 79.21$$

$$\text{PQ} = \text{tangent offset} = \text{NQ}\sin\alpha \qquad 79.22$$

The short chord distance is

$$\text{NQ} = C = 2R\sin\alpha \qquad 79.23$$

$$\text{NP} = (2R\sin\alpha)\cos\alpha$$

$$= c\cos\alpha \qquad 79.24$$

$$\text{PQ} = (2R\sin\alpha)\sin\alpha$$

$$= 2R\sin^2\alpha \qquad 79.25$$

7. CURVE LAYOUT BY CHORD OFFSET

The *chord offset method* is a third method for laying out horizontal curves. This method is also suitable for short curves. The method is named for the way in which the measurements are made, which is by measuring distances along the main chord from the instrument location at PC.

$$\text{NR} = \text{chord distance} = \text{NQ}\cos\left(\frac{I}{2} - \alpha\right)$$

$$= (2R\sin\alpha)\cos\left(\frac{I}{2} - \alpha\right)$$

$$= C\cos\left(\frac{I}{2} - \alpha\right) \qquad 79.26$$

$$\text{RQ} = \text{chord offset} = \text{NQ}\sin\left(\frac{I}{2} - \alpha\right)$$

$$= (2R\sin\alpha)\sin\left(\frac{I}{2} - \alpha\right)$$

$$= C\sin\left(\frac{I}{2} - \alpha\right) \qquad 79.27$$

8. HORIZONTAL CURVES THROUGH POINTS

Occasionally, it is necessary to design a horizontal curve to pass through a specific point. The following procedure can be used. (Refer to Fig. 79.6.)

step 1: Calculate α and m from x and y. (If α and m are known, skip this step.)

$$\alpha = \arctan\left(\frac{y}{x}\right) \qquad 79.28$$

$$m = \sqrt{x^2 + y^2} \qquad 79.29$$

step 2: Calculate γ. Since $90° + I/2 + \alpha + \gamma = 180°$,

$$\gamma = 90° - \frac{I}{2} - \alpha \qquad 79.30$$

step 3: Calculate ϕ.

$$\phi = 180° - \arcsin\left(\frac{\sin\gamma}{\cos\left(\frac{I}{2}\right)}\right) \qquad 79.31$$

step 4: Calculate θ.

$$\theta = 180° - \gamma - \phi \qquad 79.32$$

step 5: Calculate the curve radius, R, from the law of sines.

$$\frac{\sin\theta}{m} = \frac{\sin\phi\cos\left(\frac{I}{2}\right)}{R} \qquad 79.33$$

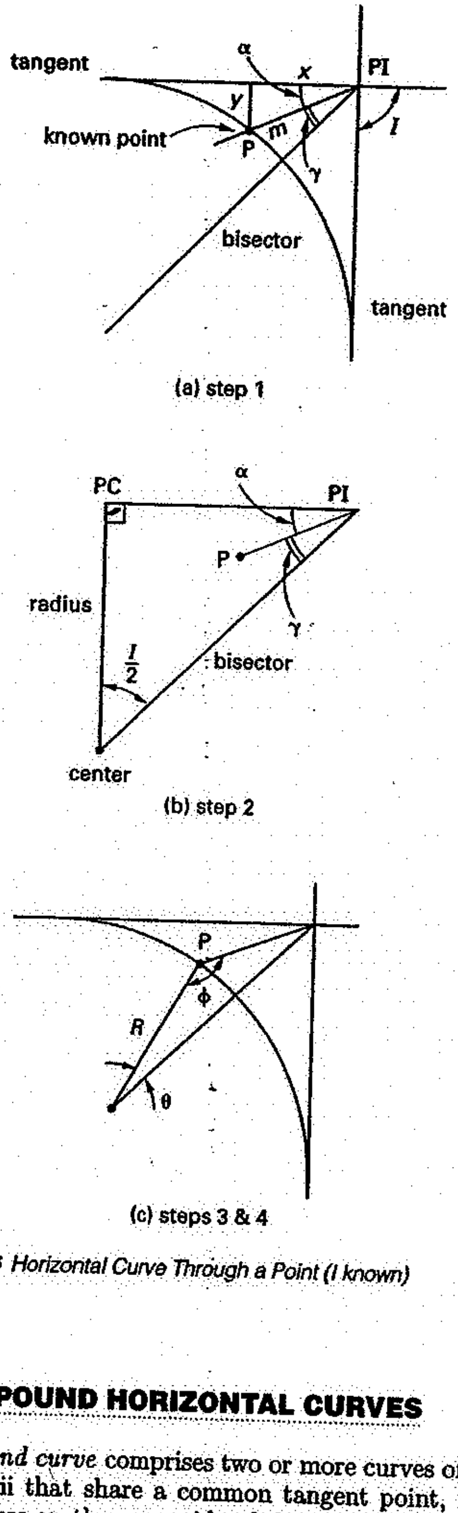


Figure 79.6 Horizontal Curve Through a Point (I known)

9. COMPOUND HORIZONTAL CURVES

A *compound curve* comprises two or more curves of different radii that share a common tangent point, with their centers on the same side of the common tangent. The PT for the first curve and the PC for the second curve coincide. This point is the *point of common curvature* (PCC).

Compound curves should only be used on low-speed roadways, such as on entrance or exit ramps. Their use is reserved for applications where design constraints, such as topography or high land cost, preclude the use of a circular or spiral curve. For safety, the radius of the larger curve should be less than or equal to four thirds times the radius of the smaller curve (three halves on interchanges).

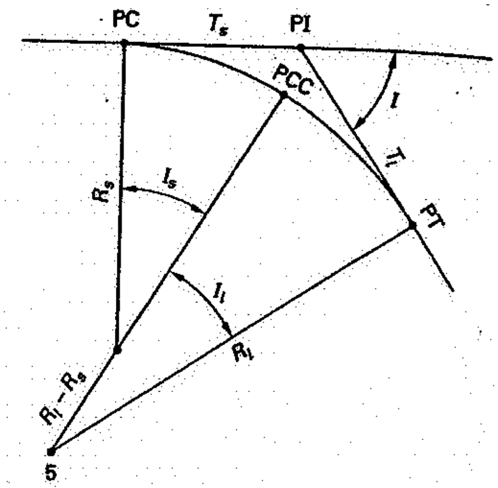


Figure 79.7 Compound Circular Curve

10. SUPERELEVATION

If a vehicle travels in a circular path with instantaneous radius R and tangential velocity (i.e., speed) v_t, it will experience an apparent centrifugal force, F_c.

$$F_c = \frac{m v_t^2}{R} \quad \text{[consistent units]}$$

The centrifugal force can be resisted by bank[illegible] tion, or a combination of the two. If it is [illegible] that banking (no friction) alone will resist the [illegible] gal force, the banking angle, ϕ, is given by E[illegible] The elevation difference between the inside an[illegible] edges of the curve is the *superelevation*. T[illegible] verse slope of the roadway has units of ft/f[illegible] hence the synonymous name of *superelevatio*[illegible] for slope. (However, runoff and transition [illegible] not the same.)

$$e = \tan\phi = \frac{v^2}{gR} \quad \text{[consistent units]}$$

Some of the centrifugal force is usually resiste[illegible] *ways friction*. (One-half of the theoretical s[illegible] tion rate is usually adequate.) Equation 79[illegible] basic formula used for determining the supe[illegible] rate when friction is relied upon to counterac[illegible] the centrifugal force. The sum, $e + f_s$, may b[illegible] to as the *centrifugal factor*.

$$e = \tan\phi = \frac{v^2}{gR} - f_s \quad \text{[consistent units]}$$

If the velocity, v, is expressed in common uni[illegible] becomes

$$e = \tan\phi = \frac{v_{\text{km/h}}^2}{127R} - f_s \quad \text{[SI]}$$

$$e = \tan\phi = \frac{v_{\text{mph}}^2}{15R} - f_s \quad \text{[U}$$

In general, a lower *banking angle* is used in urban areas than in rural areas. For arterial streets in downtown areas, the maximum superelevation rate is approximately 0.04 to 0.06. For arterial streets in suburban areas and on freeways where there is no snow or ice, the maximum superelevation rate is approximately 0.10 to 0.12. For arterial streets and freeways that experience snow and ice, the maximum should be 0.06 to 0.08.

Since the maximum superelevation rate is approximately 0.08 or 0.10, Eq. 79.35 can be used to calculate the minimum curve radius if the speed is known.

Many studies have been performed to determine values of the *side friction factor*, f_s, and most departments of transportation have their own standards. One methodology is to assume the side friction factor to be 0.16 for speeds less than 30 mph (50 km/h). Equations 79.38 or 79.39 can be used for higher speeds.

$$f_s = 0.16 - \frac{0.01(\mathrm{v}_{\mathrm{mph}} - 30)}{10} \quad [< 50 \text{ mph}] \qquad 79.38$$

$$f_s = 0.14 - \frac{0.02(\mathrm{v}_{\mathrm{mph}} - 50)}{10} \quad [50 \text{ to } 70 \text{ mph}] \qquad 79.39$$

Although the sophistication may be unwarranted, the friction factor for sideways slipping, f_s, can be differentiated from the straight-ahead friction factor. For sideways slipping, the friction factor may be referred to as the *side friction factor, lateral ratio, cornering ratio,* or *unbalanced centrifugal ratio.*

11. TRANSITIONS TO SUPERELEVATION

Transitions from crowned sections to superelevated sections should be gradual. When a curve is to be superelevated, it is first necessary to establish a rotational axis (often referred to as a "point") on the cross section. This is the axis around which the pavement will be rotated (longitudinally) to gradually change to the specified superelevated cross slope. The rotational axis ("point") is a longitudinal axis parallel to the instantaneous direction of travel. The location of the rotational point varies with the basic characteristics of the typical section. The following guidelines can be used.

(1) For two-lane and undivided highways, the axis of rotation is generally at (along) the original crown of the roadway. However, it may also be the edge of the outside or inside lane.

(2) On divided highways with relatively wide depressed medians, the axis of rotation can be at the crown of each roadway or at the edge of the lane or shoulder nearest the median of each roadway. Placing the axis at the crown results in median edges at different elevations, but it reduces the elevation differential between extreme pavement edges. If it is likely that the highway will be widened in the future, it will be desirable to rotate the pavement cross-slope about the inside lane or shoulder.

(3) On divided highways with narrow raised medians and moderate superelevation rates, the axis of rotation should be at the center of the median. If the combination of pavement width and superelevation rate results in substantial differences between pavement edge elevations, the axis of rotation should be at the edge of the lane or shoulder nearest the median of each roadway. If an at-grade crossing is located on the superelevated curve, the impact of intersecting traffic should be considered in selecting the axis of rotation.

(4) On divided highways with concrete median barriers, criterion (C) will control, except that rotation should occur at the barrier gutter when that method of rotation is selected.

For maximum comfort and safety, superelevation should be introduced and removed uniformly over a length adequate for the likely travel speeds. The total length of the *superelevation transition distance* is the sum of the tangent (crown) runout and the superelevation runoff. The following design factors should be considered in designing the superelevation transition distance.

(1) *Tangent runout*, T_R, also known as *tangent runoff* and *crown runoff*, is a gradual change from a normal crowned section to a point where the *adverse cross slope* on the outside of the curve has been removed. When the adverse cross slope has been removed, the elevation of the outside pavement edge will be equal to the centerline elevation. The inside pavement edge will be unchanged. The rate of removal is usually the same as the superelevation runoff rate (SRR).

(2) *Superelevation runoff*, L, is a gradual change from the end of the tangent runout to a cross section that is fully superelevated. The *superelevation runoff rate* (also known as the *transition rate*), SRR, is the rate at which the normal cross-slope of the roadway is transitioned to the superelevated cross-slope. The superelevation runoff rate is expressed in units of cross-slope elevation per unit width per unit length of traveled roadway.

For single lanes, a common superelevation runoff rate is 1 ft per foot of width for every 200 ft (60 m) length, expressed as 1:200. For speeds less than 50 mph, or if conditions are restrictive and if sufficient room is not available for a 1:200 transition rate, more abrupt rates may be used.

(3) Tangent runout and superelevation runoff distances may be calculated using Eqs. 79.40 and 79.41. w is the lane width, and p is the rate of cross slope.

$$T_R = \frac{wp}{\mathrm{SRR}} \qquad 79.40$$

$$L = \frac{we}{\mathrm{SRR}} \qquad 79.41$$

(4) Superelevation runoff rate on a curve may be determined by a *time rule* (e.g., "runoff shall be completed within 4 sec at the design speed") or by a *speed rule* (e.g., "3 ft of runoff for every mph of design speed"), regardless of the initial superelevation. Time and speed rules depend on the specifying agency.

(5) On circular curves, the superelevation runoff should be developed two thirds on the tangent and one third on the curve. This results in two thirds of the full superelevation at the beginning and ending of the curve. This is a compromise between placing the entire transition on the tangent section, where superelevation is not needed, and placing the transition on the curve, where full superelevation is needed.

(6) On spiral curves, the superelevation is developed entirely within the length of the spiral.

(7) All shoulders should slope away from the traveled lanes. The angular breaks at the pavement edges of the superelevated roadways should be rounded by the insertion of vertical curves. The minimum curve length in feet should be approximately numerically equal to the design speed in mph.

Example 79.4

One end of a horizontal curve on a two-lane highway has 12 ft lanes and a crown cross-slope of 0.02 ft/ft. The PC is at sta 10+00. The transition rate is 1:400, and the required superelevation is 0.04. Calculate the stationing where (a) the superelevation runoff begins, (b) the tangent runout begins, and (c) the curve is fully superelevated.

Solution

(a) Use Eqs. 79.40 and 79.41.

$$T_R = \frac{wp}{\text{SRR}} = \frac{(12\ \text{ft})\left(0.02\ \frac{\text{ft}}{\text{ft}}\right)}{\frac{1}{400}\ \frac{\text{ft}}{\text{ft}}} = 96\ \text{ft}$$

$$L = \frac{we}{\text{SRR}} = \frac{(12\ \text{ft})\left(0.04\ \frac{\text{ft}}{\text{ft}}\right)}{\frac{1}{400}\ \frac{\text{ft}}{\text{ft}}} = 192\ \text{ft}$$

The superelevation should be developed two thirds before the curve and one third after. Therefore, the superelevation begins at $2L/3$ before the PC.

$$\begin{aligned}\text{sta } 10{+}00 - \frac{2L}{3} &= \text{sta } 10{+}00 - \left(\tfrac{2}{3}\right)(192\ \text{ft}) \\ &= \text{sta } 10{+}00 - 128\ \text{ft} = \text{sta } 8{+}72\end{aligned}$$

(b) The tangent runout begins at

$$\text{sta } 8{+}72 - 96\ \text{ft} = \text{sta } 7{+}76$$

(c) The pavement should be fully superelevated at $L/3$ after the PC, at station

$$\begin{aligned}\text{sta } 10{+}00 + \frac{L}{3} &= \text{sta } 10{+}00 + \frac{192\ \text{ft}}{3} \\ &= \text{sta } 10{+}00 + 64\ \text{ft} = \text{sta } 10{+}64\end{aligned}$$

12. SUPERELEVATION OF RAILROAD LINES

The method of specifying superelevation for railroad lines is somewhat different than for roadways. The *equilibrium elevation*, E, of the outer rail relative to the inner rail is calculated from Eq. 79.42. The effective gauge, G_{eff}, is the center-to-center rail spacing.

$$E = \frac{G_{\text{eff}}\text{v}^2}{gR} \quad \text{[railroads]} \qquad 79.42$$

13. STOPPING SIGHT DISTANCE

The *stopping sight distance* is the distance required by a vehicle traveling at the design speed to stop before reaching a stationary object that has suddenly appeared in its path. Calculated stopping sight distances are the minimums that should be provided at any point on any roadway. Greater distances should be provided wherever possible.

Stopping sight distance is the sum of two dista[illegible] the distance traveled during driver perception and [illegible] time, and the distance traveled during bra[illegible] tion. The equations used to calculate sight [illegible] assume that the driver's eyes are 3.50 ft [illegible]) above the surface of the roadway. For sto[illegible] distances, it is assumed that the object bein[illegible] has a height of 0.5 ft (150 mm).

Equation 79.43 can be used to calculate the [illegible] sight distance, S, for straight-line travel on a [illegible] grade. In Eq. 79.43, the grade, G, is in decima[illegible] is negative if the roadway is downhill. The [illegible] of friction, f, is evaluated for a wet paveme[illegible]

$$S = \left(0.278\ \frac{\frac{\text{m}}{\text{s}}}{\frac{\text{km}}{\text{h}}}\right) t_p \text{v}_{\text{km/h}} + \frac{\text{v}^2_{\text{km/h}}}{254(f [illegible]} \quad \text{[SI]}$$

$$S = \left(1.47\ \frac{\frac{\text{ft}}{\text{sec}}}{\frac{\text{mi}}{\text{hr}}}\right) t_p \text{v}_{\text{mph}} + \frac{\text{v}^2_{\text{mph}}}{30(f + [illegible]}$$

"Desirable values" of stopping sight dis[illegible] listed in Table 79.2 for various design spe[illegible] tual distance is less than the design value[illegible]

79.2 AASHTO Minimum Stopping and Passing Sight Distances

design speed[a] (mi/hr (km/h))	assumed initial speed[a] (mi/hr (km/h))	minimum desired stopping sight distance[a,b] (ft (m))	assumed passing speed[a] (mi/hr (km/h))	passing sight distance two-lane highway[a,c] (ft (m))
20 (30)	20 (30)	106.7 (29.6)	30 (44)	810 (217)
25 (40)	24–25 (40)	138.5–146.5 (44.4)	xx (51)	xxx (285)
30 (50)	28–30 (47–50)	177.3–195.7 (57.4–62.8)	36 (59)	1090 (345)
35 (xx)	32–35 (xx–xx)	217.7–248.8 (xxxx-xxxx)	xx (xx)	xxxx (xxx)
40 (60)	36–40 (55–60)	267.0–313.3 (74.3–84.6)	44 (66)	1480 (407)
45 (70)	40–45 (63–70)	318.7–382.7 (94.1–110.8)	xx (74)	xxxx (482)
50 (80)	44–50 (70–80)	376.4–461.1 (112.8–139.4)	51 (80)	1840 (541)
55 (90)	48–55 (77–90)	432.0–537.8 (131.2–168.7)	xx (88)	xxxx (605)
60 (100)	52–60 (85–100)	501.5–633.8 (157.0–205.0)	57 (94)	2140 (670)
65 (xxx)	55–65 (xx–xxx)	549.4–724.0 (xxxx–xxxx)	60 (xx)	2310 (xxx)
70 (110)	58–70 (91–110)	613.1–840.0 (179.5–246.4)	64 (100)	2490 (728)
75 (120)	61–75 (98–120)	665.7–937.0 (202.9–285.6)	xx (106)	xxxx (792)

[a]"xxxx" indicates that the values are absent in the 1990 edition tables. "(xxxx)" indicates that the values are absent in the 1994 edition tables. For these situations, values must be determined graphically, by equation, or by interpolation.
[b]wet pavement
[c]Values should be increased by 18% for downgrades steeper than 3% and longer than 1 mi (1.609 km) in length.

is known as a "minimum value." The minimum speed to be used is defined by Eq. 79.44.

$$v_{min} = v_{design} - 0.2(v_{design} - 30 \text{ km/h}) \quad \text{[SI]} \qquad 79.44(a)$$

$$v_{min} = v_{design} - 0.2(v_{design} - 20 \text{ mph}) \quad \text{[U.S.]} \qquad 79.44(b)$$

14. PASSING SIGHT DISTANCE

The *passing sight distance* is the length of open roadway ahead necessary to pass without meeting an oncoming vehicle. Passing sight distance is applicable only to two-lane, two-way highways. Passing sight distance is not relevant on multilane highways.

Passing sight distances assume that the driver's eyes are 3.50 ft (1.07 m) above the surface of the roadway. The object being viewed is assumed to be at a height of 4.25 ft (1.30 m). Minimum passing sight distances are given in Table 79.2.

15. MINIMUM HORIZONTAL CURVE LENGTH FOR STOPPING DISTANCE

A horizontal circular curve is shown in Fig. 79.8. Obstructions along the inside of curves, such as retaining walls, cut slopes, trees, buildings, and bridge piers, can limit the available sight distance. Often, a curve must be designed that will simultaneously provide the required stopping sight distance while maintaining a clearance from a roadside obstruction.

The equations for calculating the geometry of a horizontal curve to see around an obstruction assume that the stopping sight distance is greater than the curve length (i.e., $S \geq L$). The stopping sight distance and length along the curve are the same. The angles are given in degrees, not radians.

$$S = \left(\frac{R}{28.65}\right)\left(\arccos\left(\frac{R-M}{R}\right)\right) \qquad 79.45$$

$$M = R(1 - \cos\theta) = R\left(1 - \cos\left(\frac{DS}{200}\right)\right)$$

$$= R\left(1 - \cos\left(\frac{28.65S}{R}\right)\right) \qquad 79.46$$

Decision sight distance should be added on a horizontal curve whenever a driver is confronted with additional information that may unduly complicate the highway information the driver must process. 10 sec of decision time is considered to be the minimum.

The curve length methods presented in this section are based on horizontal tangent grades only. If a vertical grade occurs in conjunction with a circular curve, these methods cannot be used.

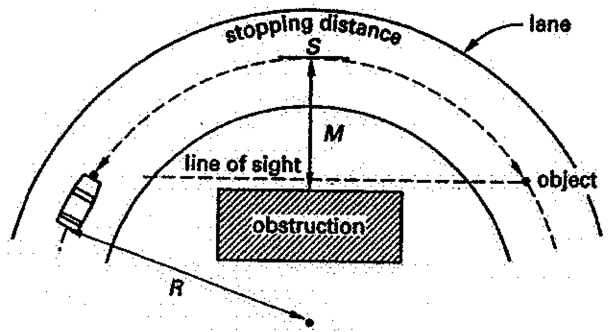


Figure 79.8 Horizontal Curve with Obstructions

16. VERTICAL CURVES

Vertical curves are used to change the grade of a highway. Most vertical curves take the shape of an *equal-tangent parabola*. Such curves are symmetrical about the vertex. Since the grades are very small, the actual arc length of the curve is approximately equal to the chord length BVC-EVC. Table 79.3 lists the standard abbreviations used to describe geometric elements of vertical curves.

A vertical parabolic curve is completely specified by the two grades and the curve length. Alternatively, the *rate of grade change per station*, R, can be used in place of curve length. The rate of grade change per station is given by Eq. 79.47. Units of %/sta are the same as ft/(sta)^2.

$$R = \frac{G_2 - G_1}{L} \quad \text{[may be negative]} \qquad 79.47$$

Equation 79.48 defines an equal-tangent parabolic curve. x is the distance to any point on the curve, measured in stations beyond the BVC, and elev is measured in ft. The same reference point is used to measure all elevations.

$$\text{elev}_x = \left(\frac{R}{2}\right)x^2 + G_1 x + \text{elev}_{\text{BVC}} \qquad 79.48$$

The maximum or minimum elevation will occur when the slope is equal to zero, which is located as determined by Eq. 79.49. This point is known as the *turning point*. In sag vertical curves, the turning point is the location at which catch basins should be installed.

$$x_{\text{turning point}} = \frac{-G_1}{R} \quad \text{[in stations]} \qquad 79.49$$

The *middle ordinate distance* is found from Eq. 79.50.

$$M_{\text{ft}} = \frac{AL_{\text{sta}}}{8} \qquad 79.50$$

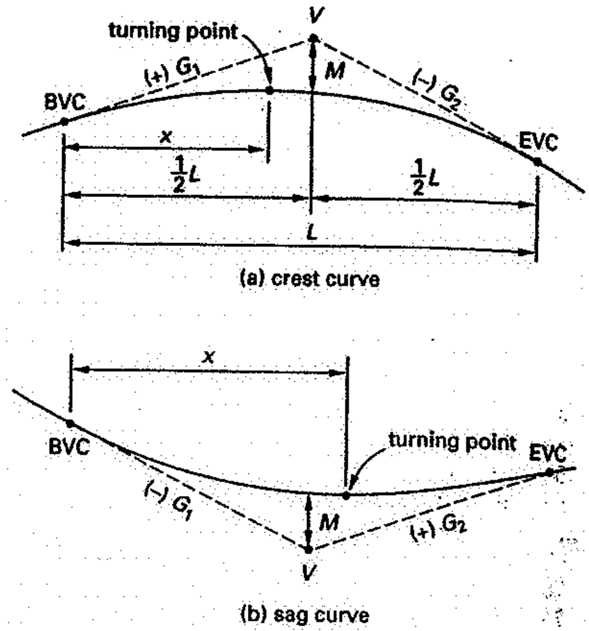


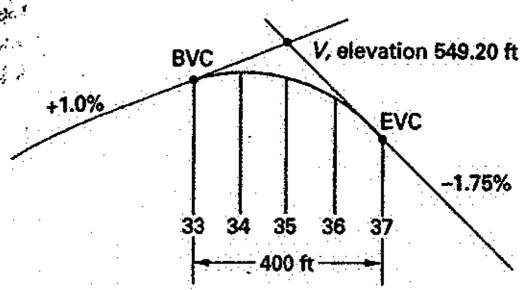
Figure 79.9 Symmetrical Parabolic Vertical Curve

Table 79.3 Vertical Curves: Abbreviations and Terms

preferred	
A	change in gradient, $\lvert G_2 - G_1 \rvert$ [always po
BVC	beginning of the vertical curve
EVC	end of the vertical curve
G_1	grade from which the stationing starts, in percent
G_2	grade toward which the stationing head in percent
L	length of curve
M	middle ordinate
R	rate of change in grade per station
V	vertex (the intersection of the two tang
alternate/archaic	
E	tangent offset at V (same as M)
EVT	end of vertical tangency (same as EVC and PVT)
PVC	same as BVC
PVI	same as V
PVT	same as EVC
VPI	vertical point of intersection (same as

Example 79.5

A crest vertical curve with a length of 400 grades of +1.0% and −1.75%. The vertex is station 35+00 and elevation 549.20 ft. W elevations of the (a) BVC, (b) EVC, and stations on the curve?



Solution

The curve length is 4 stations (400 ft).

(a)
$$\text{elev}_{\text{BVC}} = \text{elev}_{\text{V}} - G_1\left(\frac{L}{2}\right)$$
$$= 549.20 \text{ ft} - \left(1 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{4 \text{ sta}}{2}\right)$$
$$= 547.20 \text{ ft}$$

(b)
$$\text{elev}_{\text{EVC}} = \text{elev}_{\text{V}} - G_2\left(\frac{L}{2}\right)$$
$$= 549.20 \text{ ft} - \left(1.75 \ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{4 \text{ sta}}{2}\right)$$
$$= 545.70 \text{ ft}$$

(c) Use Eq. 79.47.
$$R = \frac{G_2 - G_1}{L}$$
$$= \frac{-1.75\% - 1\%}{4 \text{ sta}}$$
$$= -0.6875 \ \%/\text{sta}$$
$$\frac{R}{2} = \frac{-0.6875 \frac{\%}{\text{sta}}}{2}$$
$$= -0.3438\%/\text{sta} \quad [\text{same as } -0.3438 \text{ ft/sta}^2]$$

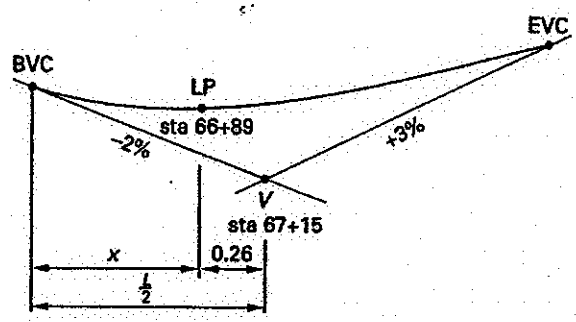
The equation of the curve is
$$\text{elev}_x = \left(\frac{R}{2}\right)x^2 + x + \text{elev}_{\text{BVC}}$$
$$= -0.3438x^2 + x + 547.20 \text{ ft}$$

At sta 34+00, $x = 34 - 33 = 1$ sta.
$$\text{elev}_{34+00} = \left(-0.3438 \ \frac{\text{ft}}{\text{sta}^2}\right)(1 \text{ ft})^2 + 1 \text{ ft} + 547.20 \text{ ft}$$
$$= 547.86 \text{ ft}$$

Similarly,
$$\text{elev}_{35+00} = \left(-0.3438 \ \frac{\text{ft}}{\text{sta}^2}\right)(2 \text{ ft})^2 + 2 \text{ ft} + 547.20 \text{ ft}$$
$$= 547.82 \text{ ft}$$
$$\text{elev}_{36+00} = \left(-0.3438 \ \frac{\text{ft}}{\text{sta}^2}\right)(3 \text{ ft})^2 + 3 \text{ ft} + 547.20 \text{ ft}$$
$$= 547.11 \text{ ft}$$

Example 79.6

A vertical sag curve with vertex located at sta 67+15 has a low point at sta 66+89. The grade into the curve is −2%, and the grade out of the curve is +3%. What is the length of curve?



Solution

The location of the low point is defined by Eqs. 79.47 and 79.49.

$$R = \frac{G_2 - G_1}{L} = \frac{3\frac{\%}{\text{sta}} - \left(-2\frac{\%}{\text{sta}}\right)}{L} = \frac{5\frac{\%}{\text{sta}}}{L}$$
$$x = \frac{-G_1}{R} = \frac{-\left(-2\frac{\%}{\text{sta}}\right)}{\frac{5\frac{\%}{\text{sta}}}{L}}$$
$$= 0.4L$$

The distance between the low point and the vertex is 0.26 sta. The distance between BVC and the vertex is

$$\frac{L}{2} = x + 0.26 \text{ sta}$$

Substituting $x = 0.4L$,

$$\frac{L}{2} = 0.4L + 0.26 \text{ sta}$$
$$L = 2.6 \text{ sta} \quad (260 \text{ ft})$$

17. VERTICAL CURVES THROUGH POINTS

If a curve is to have some minimum clearance from an obstruction as shown in Fig. 79.10, the curve length generally will not be known in advance. If the station and elevation of the point, P, the station and elevation of the BVC or the vertex, and the gradient values G_1 and G_2 are known, the curve length can be determined explicitly.

step 1: Find the elevation of points E, F, and G.

step 2: Calculate the constant (no physical significance) s.

$$s = \sqrt{\frac{\text{elev}_{\text{E}} - \text{elev}_{\text{G}}}{\text{elev}_{\text{E}} - \text{elev}_{\text{F}}}} \qquad 79.51$$

step 3: Solve for L directly.

$$L = \frac{2d(s+1)}{s-1} \qquad 79.52$$

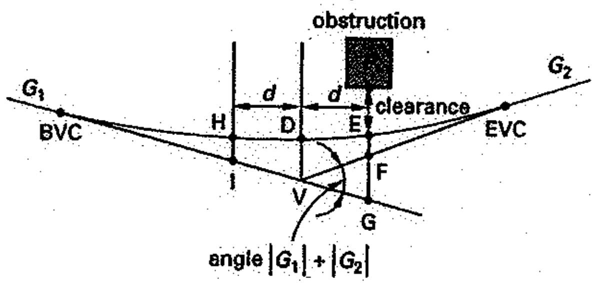


Figure 79.10 Vertical Curve with an Obstruction

Example 79.7

Determine the length of a sag vertical curve that passes through a point at elevation 614.00 ft and sta 17+00. The grades are $G_1 = -4.2\%$ and $G_2 = +1.6\%$. The BVC is at sta 13+00, elevation = 624.53 ft.

Solution

The distance from the BVC to the point is

$$\begin{aligned} x &= 17 \text{ sta} - 13 \text{ sta} \\ &= 4 \text{ sta} \end{aligned}$$

At the given point for this curve,

$$\text{elev} = 614.00 \text{ ft}$$

$$R = \frac{G_2 - G_1}{L} = \frac{1.6\frac{\%}{\text{sta}} - \left(-4.2\frac{\%}{\text{sta}}\right)}{L} = \frac{5.8\frac{\%}{\text{sta}}}{L}$$

The equation of the curve is

$$\text{elev} = \frac{Rx^2}{2} + G_1 x + \text{elev}_{\text{BVC}}$$

$$614.00 \text{ ft} = \frac{\left(\frac{5.8\frac{\%}{\text{sta}}}{L}\right)(4 \text{ sta})^2}{2} + \left(-4.2\frac{\text{ft}}{\text{sta}}\right)(4 \text{ sta}) + 624.53 \text{ ft}$$

$$-10.53 \text{ ft} = \frac{46.4}{L} - 16.8 \text{ ft}$$

$$L = 7.4 \text{ sta} \quad (740 \text{ ft})$$

18. VERTICAL CURVE TO PASS THROUGH TURNING POINT

Another common situation involving equal-tangent vertical curves is calculating the length of a sag or crest curve needed to pass through a turning point, TP, at a particular elevation. Equation 79.53 can be used to determine the curve length directly. Grades G_1 and G_2 and the elevations of the PVI and the turning point must be known.

$$L = \frac{2(\text{elev}_{\text{V}} - \text{elev}_{\text{TP}})}{G_1\left(\frac{G_1}{G_2 - G_1} + 1\right)} \qquad 79.53$$

19. MINIMUM VERTICAL CURVE LENGTH FOR SIGHT DISTANCES (CREST CURVES)

Crest curve lengths are generally determined based on stopping sight distances. (Headlight sight distance and rider comfort control the design of sag vertical curves.) The passing sight distance could also be used, except that the required length of the curve would be much greater than that based only on the stopping sight distance. Since the curve length determines the extent of the earthwork required, and it is easier (i.e., less expensive) to prohibit passing on crest curves than to perform the earthwork required to achieve the required passing sight distance, only the stopping sight distance is usually considered in designing the curve length.

Two factors affect the sight distance: the algeb[illegible] ference, A, between gradients of the intersect[illegible] gents, and the length of the vertical curve, L[illegible] small algebraic difference in grades, the leng[illegible] vertical curve may be short. However, to o[illegible] same sight distance with a large algebraic di[illegible] grades, a much longer vertical curve is neede[illegible]

Table 79.4 implies that the choice of a curve [illegible] simple selection of sight distance based on de[illegible] This is actually the case in simple curve len[illegible] problems where the design speed, v, and [illegible] and G_2, are known. The required stopping [illegible] determined from Table 79.2 or calculated fro[illegible] or 79.45. The required curve length, L, is [illegible] from the formulas in Table 79.4. Since it [illegible] unknown, curve length is calculated for both [illegible] and $S > L$ cases. The calculated curve len[illegible] inconsistent with its assumption is discarde[illegible]

The constants in Table 79.4 are based on spe[illegible] of objects and driver's eyes above the road[illegible] general, the sight distance over the crest o[illegible] curve is given by Eqs. 79.54 and 79.55, whe[illegible] height of the eyes of the driver and h_2 is th[illegible] the object sighted, both in feet.

$$L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} \quad [S < L]$$

$$L = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A} \quad [S \ldots$$

The sight distance under an overhead structure to see an object beyond a sag vertical curve is given by Eqs. 79.56 and 79.57. Various assumptions, including headlight height and beam divergence, were made by AASHTO in developing the following equations.

$$L = \frac{AS^2}{120 + 3.5S} \quad [S < L] \quad [\text{SI}] \qquad 79.56(a)$$

$$L = \frac{AS^2}{400 + 3.5S} \quad [S < L] \quad [\text{U.S.}] \qquad 79.56(b)$$

$$L = 2S - \frac{120 + 3.5S}{A} \quad [S > L] \quad [\text{SI}] \qquad 79.57(a)$$

$$L = 2S - \frac{400 + 3.5S}{A} \quad [S > L] \quad [\text{U.S.}] \qquad 79.57(b)$$

Table 79.4 AASHTO Required Lengths of Curves on Grades[a]

	stopping sight distance[b] (crest curves)	passing sight distance (crest curves)	stopping sight distance (sag curves)
SI units			
$S < L$	$\frac{AS^2}{404}$	$\frac{AS^2}{946}$	$\frac{AS^2}{120 + 3.5S}$
$S > L$	$2S - \frac{404}{A}$	$2S - \frac{946}{A}$	$2S - \frac{120 + 3.5S}{A}$
U.S. units			
$S < L$	$\frac{AS^2}{1329}$	$\frac{AS^2}{3093}$	$\frac{AS^2}{400 + 3.5S}$
$S > L$	$2S - \frac{1329}{A}$	$2S - \frac{3093}{A}$	$2S - \frac{400 + 3.5S}{A}$

[a] $A = G_1 - G_2$, the algebraic difference in grades.

[b] The driver's eye is 3.50 ft (1070 mm) above road surface, viewing an object 0.5 ft (150 mm) high.

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Example 79.8

A car is traveling up a 1.25% grade of a crest curve with a design speed of 40 mph. The descending grade is −2.75%. What is the required length of curve for minimum proper stopping sight distance?

$$A = |G_2 - G_1| = |-2.75 - 1.75| = 4.0$$

Solution

Table 79.2 gives the minimum stopping sight distance at 40 mph as 267.0 ft. Refer to Table 79.4. Assume $S > L$.

$$L = 2S - \frac{1329}{A}$$
$$= (2)(267.0\ \text{ft}) - \frac{1329}{4.0} = 201.8\ \text{ft}$$

Using Table 79.4 and assuming $S < L$,

$$L = \frac{AS^2}{1329}$$
$$= \frac{(4.0)(267.0\ \text{ft})^2}{1329} = 214.6\ \text{ft}$$

Since 267.0 ft is greater than 214.6 ft ($S > L$), the second assumption is not valid. The required curve length is 201.8 ft.

20. DESIGN OF CREST CURVES USING K-VALUE

AASHTO extends the sight-distance procedure by considering the design speed to be a nominal value. The actual speed of a vehicle might be higher or lower. If the actual speed is equal to the design speed, travel is considered to be in the "upper range of speeds." Alternatively, the vehicle may be assumed to be driven somewhat more slowly than the design speed to compensate for weather, wet pavement, lighting, or other adverse conditions. This case is referred to as the "lower range of speeds." (AASHTO reports that studies have concluded that vehicles are not always actually driven more slowly under these conditions, but the assumption may yet be made.) The normally used upper- and lower-range speed values are listed as the "initial speed" values in Table 79.2.

The *K-value method* of analysis used in AASHTO's Green Book is a simplified method of choosing a stopping sight distance for a crest vertical curve. The length of vertical curve per percent grade difference, K, is the ratio of the curve length, L, to grade difference, A.

$$K = \frac{L}{A} = \frac{L}{|G_2 - G_1|} \quad [\text{always positive}] \qquad 79.58$$

The $L = KA$ relationship is conveniently linear. In order to facilitate rapid calculation of curve lengths, AASHTO has prepared several graphs. Figure 79.11 gives minimum lengths of crest vertical curves for the upper speed range, and Fig. 79.12 gives minimum curve lengths for the lower speed range. Since for a fixed grade difference the speed determines the stopping distance, every value of speed has a corresponding value of K. Thus, the curves in the figures are identified concurrently with the speed and the K-value. It is not necessary to specify both A and L in a design problem, as knowing K is sufficient.

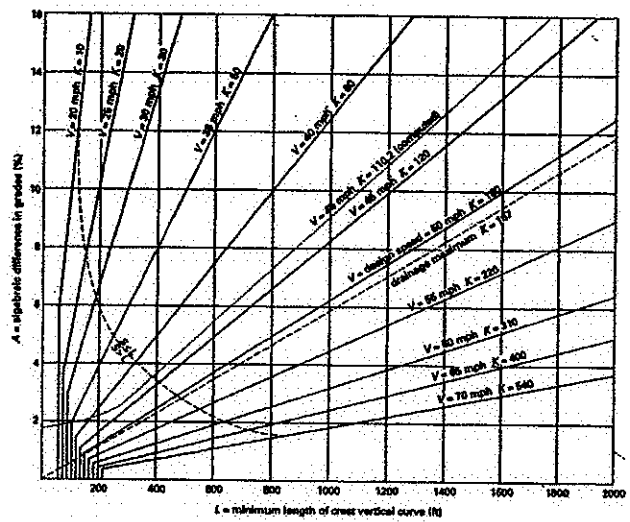

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Figure 79.11 Design Controls for Crest Vertical Curves (upper range)

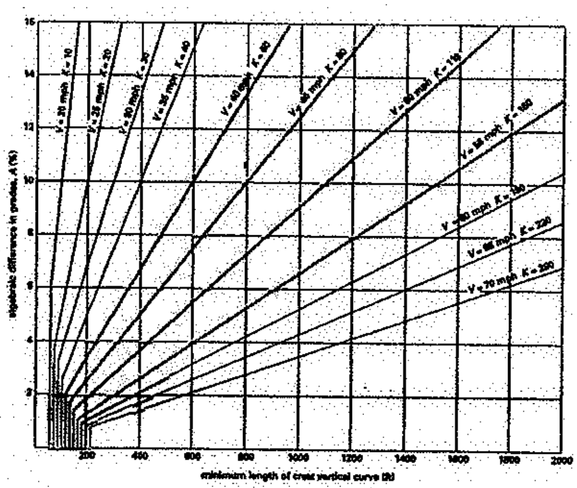

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Figure 79.12 Design Controls for Crest Vertical Curves (lower range)

The simplified procedure is to select one figure or another based on upper or lower speed range, then select one of the curves based on the speed or the K-value, and finally read the curve length that corresponds to the grade difference, A. K-values shown on the graphs have been rounded for design.

The shorter curve lengths in the AASHTO figure have been determined by other overriding factors, the most important of which are experience and state requirements. Curve lengths calculated from Table 79.4 for $S > L$ often do not represent desirable design practice and are replaced by estimated values of three times the design speed. This is consistent with the minimum curve lengths of 100 to 300 ft (30 to 90 m) prescribed by most states. The estimated solutions in the AASHTO figures are also justified on the basis that the longer curve lengths are obtained inexpensively when the difference in grades is small.

21. MINIMUM VERTICAL CURVE LENGTH FOR HEADLIGHT SIGHT DISTANCE: SAG CURVES

A *sag curve* should be designed so that a vehicle's headlights will illuminate a minimum distance of road ahead equal to the stopping sight distance. When full roadway lighting is available and anticipated to be available for the foreseeable future, designing for the *headlight sight distance* (also known as *light distance*) may not be necessary.

22. MINIMUM VERTICAL CURVE LENGTH FOR COMFORT: SAG CURVES

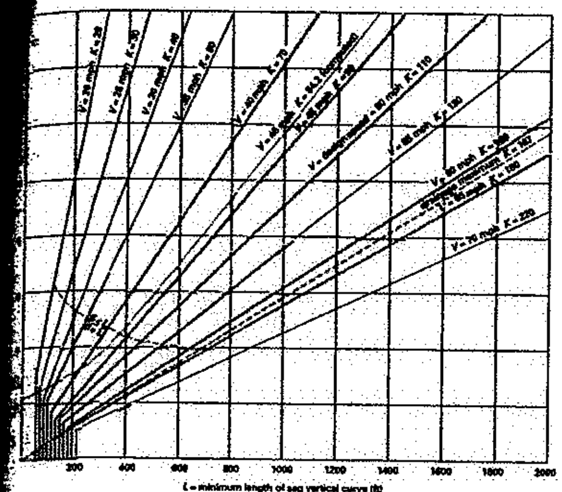
In a sag curve, gravitational and centrifugal forces combine to simultaneously act on the driver and passengers and comfort becomes the design control. A formula for calculating the minimum length of curve for comfort is given in Eq. 79.59. The curve length for comfort may be shorter or longer than the safe passing, [illegible] sight distances.

$$L = \frac{Av_{km/h}^2}{395} \quad \text{[SI]}$$

$$L = \frac{Av_{mph}^2}{46.5} \quad \text{[U.S.]}$$

23. DESIGN OF SAG CURVES US[ING] K-VALUE

The AASHTO Green Book contains a grap[h...] for determining the minimum lengths o[f...] curves also. Factors taken into consider[ation...] headlight sight distance, rider comfort, [...] trol, and rules of thumb. Figures 79.13 & [...] vide graphical methods of determining th[e...] the K-value concept. As with crest cu[rves...] dure is to select one figure or another b[ased...] lower speed range, then select one of the c[urves...] the speed or the K-value (from Eq. 79.[...]) read the curve length that corresponds t[o...] ference, A.



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79.13 *Design Controls for Sag Vertical Curves (upper range)*

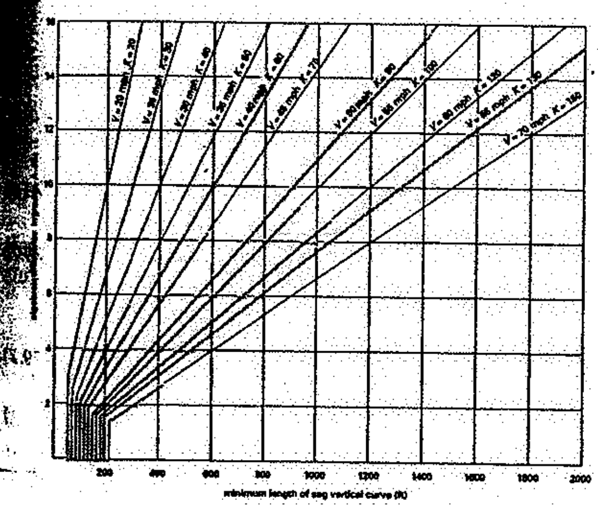

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Figure 79.14 *Design Controls for Sag Vertical Curves (lower range)*

24. UNEQUAL TANGENT (UNSYMMETRICAL) VERTICAL CURVES

Not all vertical curves are symmetrical about their vertices. Figure 79.15 illustrates a curve in which the distance from the BVC to the vertex, l_1, is not the same as the distance from the vertex to the EVC, l_2. To evaluate the curve, a line v_1v_2 is drawn parallel with AB such that $Av_1 = v_1V$ and $Vv_2 = v_2B$. This line divides the curve into two halves of equal-tangent parabolic vertical curves: the first from A to K, and the second from K to B. The elevation of v_1 is the average of that of A and V; the elevation of v_2 is the average of that of V and B. Therefore, CK = KV. The vertical distance, CV, is defined by Eq. 79.27.

$$\text{CV} = \left(\frac{l_1 l_2}{L}\right) A \qquad 79.60$$

In Eq. 79.60, l_1, l_2, and L are expressed in stations, and A is expressed in percent. Eq. 79.61 can be used to solve for the distance CK.

$$\text{CK} = \text{KV} = \left(\frac{l_1 l_2}{2L}\right) A \qquad 79.61$$

The grade, G', of the line v_1v_2 is the same as that of AB, found by Eq. 79.62.

$$G' = \frac{\text{elev}_\text{B} - \text{elev}_\text{A}}{L} \qquad 79.62$$

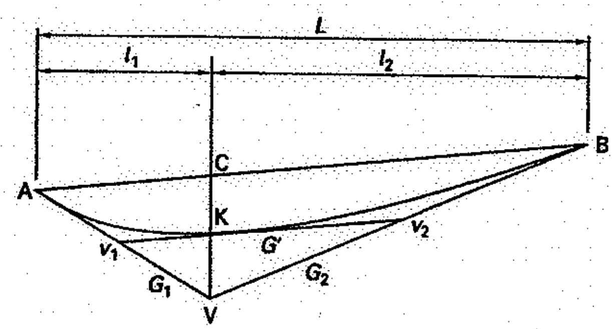


Figure 79.15 *Unequal Tangent Vertical Curve*

25. SPIRAL CURVES

Spiral curves (also known as *transition curves* and *easement curves*) are used to produce a gradual transition from tangents to circular curves. A spiral curve is a curve of gradually changing radius and gradually changing degree of curvature. Figure 79.16 illustrates the geometry of spiral curves connecting tangents with a circular curve of radius R and degree of curvature D. The *entrance spiral* begins at the left at the TS (tangent to spiral) and ends at the SC (spiral to curve). The circular curve begins at the SC and ends at the CS (curve to spiral). The *exit spiral* begins at the CS and ends at the ST (spiral to tangent).

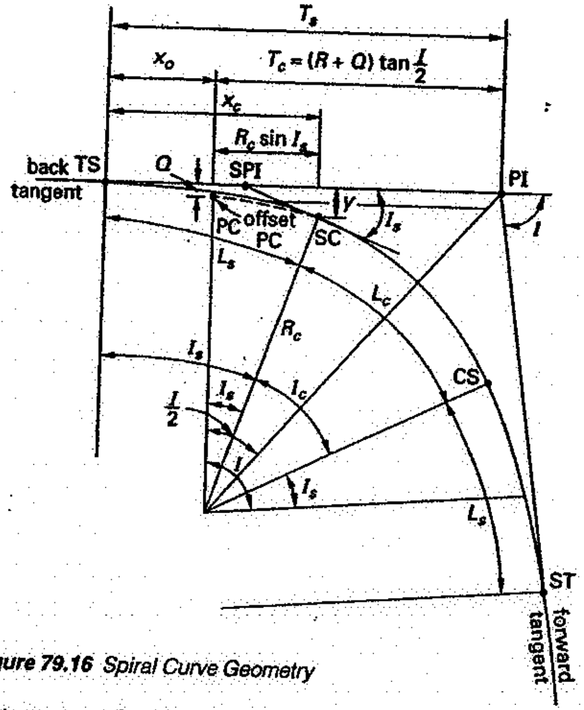


Figure 79.16 Spiral Curve Geometry

Entrance and exit spirals are geometrically identical. Their length, L_s, is the arc distance from the TS to the SC (or CS to ST). This length is selected to provide sufficient distance for introducing the curve's superelevation. The length of a spiral curve can be adjusted to between 75% and 200% of the theoretical value to meet other design criteria.

There are various ways of selecting the spiral length, L_s, including rules of thumb, tables, formulas, and codes. Equation 79.63 is a modification of the 1909 Shortt equation.

$$L_{s,\mathrm{m}} \approx \frac{0.035\mathrm{v}_{\mathrm{kph}}^3}{R_{\mathrm{m}}} \quad \text{[SI]} \qquad 79.63(a)$$

$$L_{s,\mathrm{ft}} \approx \frac{1.6\mathrm{v}_{\mathrm{mph}}^3}{R_{\mathrm{ft}}} \quad \text{[U.S.]} \qquad 79.63(b)$$

A tangent to the entrance spiral at the SC projected to the back tangent locates the SPI (spiral point of intersection). The angle at the SPI between the two tangents is the *spiral angle*, I_s. (As with spiral curves, the symbol I and Δ are both used for spiral angle.) The spiral's degree of curvature changes uniformly from 0° at TS to D (the circular curve's degree) at SC. Since the change is uniform, the average degree of curve over the spiral's length is $D/2$. The spiral angle is given by Eq. 79.64. L_s is the length of spiral curve in ft, and I_s, I_c, and D are in degrees.

$$I_s = \left(\frac{L_s}{100}\right)\left(\frac{D}{2}\right) = \frac{L_s D}{200} \qquad 79.64$$

The *total deflection angle* is equal to the interior angle, I, of the two intersecting tangents.

$$I = I_c + 2I_s \qquad 79.65$$

The *total length of the curve system* is

$$L = L_c + 2L_s \qquad 79.66$$

As with circular curves, spiral curves can be laid out using deflection angles or tangent offsets. The total deflection angle, α_s, from the TS to the SC is

$$\alpha_s = \frac{y}{x} \approx \frac{I_s}{3} \qquad 79.67$$

At any other point, P, along the spiral curve, spiral angles, α_{P}, are proportional to the square of the distance from the TS to the point. I_{P} is the central spiral angle at any point P whose distance from TS is L_{P}.

$$\frac{\alpha_{\mathrm{P}}}{\alpha_s} = \frac{I_{\mathrm{P}}}{I_s} = \left(\frac{L_{\mathrm{P}}}{L_s}\right)^2 \qquad 79.68$$

For any spiral curve length L up to L_s, the tangent offset is

$$\begin{aligned} y &= x\tan\alpha_{\mathrm{p}} \approx x\alpha_{\mathrm{p,radians}} \\ &= \frac{xI_{s,\mathrm{radians}}}{3} \\ &= \frac{xL_sD}{(200)(3)} \\ &= \frac{xL_s}{6R} \quad [I < 20°] \end{aligned} \qquad 79.69$$

$$T_c = (R+Q)\tan\frac{I}{2} \qquad 79.70$$

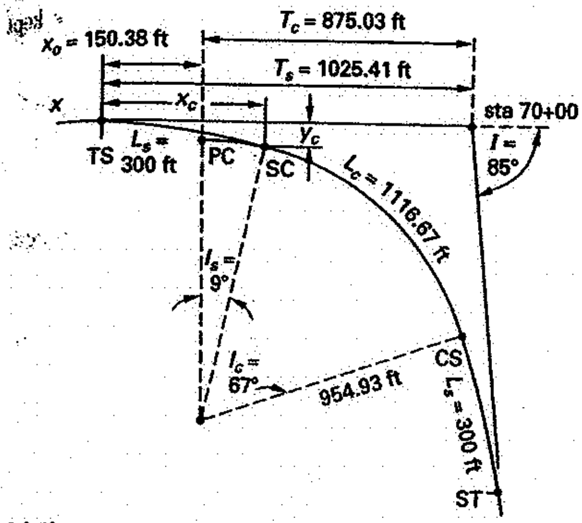
$$\begin{aligned} Q &= y - R(1-\cos I_s) \\ &= \frac{L_s^2}{6R} - R(1-\cos I_s) \end{aligned}$$

Table 79.5 Spiral Curve Abbreviations

CS	curve to spiral point
I_s	spiral angle
L	chord length
L_{P}	distance to any point, P
L_s	length of spiral
Q	offset of ghost PC to new tangent (tange
R	radius of circular curve
SC	spiral to curve point
SPI	spiral to point of intersection
ST	spiral to tangent
TS	tangent to spiral point

Example 79.9

A 300 ft long spiral curve is used as a transi circular curve. The interior angle of the tan The station of the PI is 70+00. Determine of the (a) TS, (b) SC, (c) CS, and (d) ST,



Solution

Use Eq. 79.64.

$$I_s = \frac{L_s D}{200} = \frac{(300\text{ ft})(6°)}{200} = 9°$$

The circular curve's subtended angle between the SC and CS is

$$I_c = I - 2I_s = 85° - (2)(9°) = 67°$$

Use Eq. 79.1 to calculate the circular curve's radius.

$$R_c = \frac{5729.578}{D_c} = \frac{5729.578}{6°} = 954.93\text{ ft}$$

Use Eq. 79.10 to calculate the length of the circular curve.

$$L_c = 100\left(\frac{I_c}{D_c}\right) = (100)\left(\frac{67°}{6°}\right) = 1116.67\text{ ft}$$

Use Eq. 79.69 to calculate the offset from the tangent at the SC. At that point, $x \approx L_s$.

$$y = \frac{L_s^2}{6R_c} = \frac{(300\text{ ft})^2}{(6)(954.93\text{ ft})} = 15.71\text{ ft}$$

Since $9° < 20°$, the tangent distance can be approximated as

$$x_c = \frac{y_c}{\tan\left(\frac{I_s}{3}\right)} = \frac{15.71\text{ ft}}{\tan\left(\frac{9°}{3}\right)} = 299.76\text{ ft}$$

From Fig. 79.16,

$$x_o = x_c - R_c \sin I_s = 299.76\text{ ft} - (954.93\text{ ft})(\sin 9°) = 150.38\text{ ft}$$

Use Eq. 79.70.

$$Q = \frac{L_s^2}{6R} - R(1 - \cos I_s) = \frac{(300\text{ ft})^2}{(6)(954.93\text{ ft})} - (954.93\text{ ft})(1 - \cos 9°) = 3.95\text{ ft}$$

Use Eq. 79.70.

$$T_c = (R_c + Q)\tan\left(\frac{I}{2}\right) = (954.93\text{ ft} + 3.95\text{ ft})\tan\left(\frac{85°}{2}\right) = 878.65\text{ ft}$$

$$T_s = x_o + T_c = 150.38\text{ ft} + 878.65\text{ ft} = 1029.03\text{ ft}$$

(a) sta TS = sta PI − T_s = (70+00) − 1029.03 ft = 59+70.97

(b) sta SC = sta TS + L_s = (59+70.97) + 300 ft = 62+70.97

(c) sta CS = sta SC + L_c = (62+70.97) + 1116.67 ft = 73+87.64

(d) sta ST = sta CS + L_s = (73+87.64) + 300 ft = 76+87.64

26. AIRPORT PAVEMENT GRADES

Longitudinal grades of airport runways should be limited to 1.5% for transport airports and 2.0% for utility airports. Similarly, the maximum grade change from one runway grade to another grade should be limited to 1.5% for transport airports and 2.0% for utility airports. The maximum grade on the first and last quarter of the runway distance should be 0.8% for transport airports. The minimum lengths of vertical curves (PC to PT) should be 1000 ft/% grade change for transport airports and 300 ft/% grade change for utility airports. The minimum separation between points of intersection (PIs) of runway vertical curves should be $(1000\text{ ft})(|G_1| + |G_2|)$ for transport airports and $(250\text{ ft})(|G_1|+|G_2|)$ for utility airports.

27. RAILROAD GRADES

Table 79.6 gives general recommendations for maximum rates of change of gradients for railroad curves.

Table 79.6 *Maximum Rate of Change of Gradient on Railroad Lines (percent per 100 ft station)*

	curve type	
track line rating	sag	crest
high-speed main line	0.05	0.10
secondary or branch line	0.10	0.20